Conceptual Misunderstanding in Senior High School Algebra among Senior High School Mathematics Teachers', Prospective Teachers' and Students

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Abstract: This study aimed at exploring Senior High School Mathematic Teachers, Prospective Teachers and Student's conceptual misunderstanding on Senior High School algebra with an intent to uncover the errors they make as a result of conceptual misunderstanding. A test consisting of fourteen (14) tasks was used for data collection. A sample of 210 consisting of sixty (60) prospective senior high school mathematics Teachers from mathematics education department of University of Education Winneba forty (40) SHS mathematics teachers and one hundred and ten (110) senior high school students from four (4) selected senior high schools in Ashanti region of Ghana. The study employed convenience, purposive and simple random sampling as sampling techniques and descriptive survey design as the research design. The data collection tools used were test and semi structured interview guide. Constructivism and behaviourism theories were employed as the theoretical frame work for the study. The study identified seven (7) categories of conceptual misunderstanding in Senior High School algebra among the prospective teachers and the students' whiles six of these seven were also found among the teachers. The seven conceptual misunderstanding identified were on algebraic variables, algebraic expressions, algebraic equations and algebraic word problems. The study recommends that teachers, prospective teachers and students should be aware of the existence of conceptual misunderstanding in teaching and learning of algebraic concept. The study also recommends that, heads of schools should organize workshops and refresher courses for mathematics teachers on sensitive topics like conceptual misunderstandings in mathematics.

Keywords: Conceptual Misunderstanding, Algebra, High School, Mathematics

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Introduction

The relevance of mathematics in one's life as an individual, society and national development cannot be undermined (Fletcher, 2016). According to Fletcher (2016), difficulty in mathematics is a serious issue. Success or failure in mathematics at school has a decisive influence on the choice of further education and career. Algebra is a branch of mathematics that is one of the major areas covered to enhance the acquisition of mathematical knowledge. Difficulty in algebra affects performance in mathematics since algebra is said to be the mother of mathematics. There have been several studies (internationally, continentally and locally) on errors and conceptual misunderstanding on algebra (Olivier, 1989; Allen, 2007; Egodawatte, 2011; Makonya, 2011; Chamundeswari, 2014; Makhubele, 2014; Mutunge 2016; Adu, Asuah & Asiedu-Addo, 2015; Adu, 2016; Bintu 2018), these studies aimed at addressing the issue of errors and conceptual misunderstanding in algebra that affect mathematics performance. Most of these studies focused on only students alone and very few were on prospective teachers (on pedagogical content knowledge) and on teachers (teaching methods). They also considered one or two aspects of algebra (i.e., either on algebraic variable or, expression, or equation or in word problem).

Allen (2007) in students thinking found that students' conceptual misunderstanding in algebra is one facet of mathematics in general. Certain conceptual relations that are acquired may be inappropriate within a certain context. Such relations are termed as conceptual misunderstandings and these may be due to inaccurate or incorrect thinking. In turn, student conceptual misunderstandings cause teachers' immense frustration about why their teaching isn't getting through. Conceptual misunderstandings, once rooted in the student's memory, are hard to ease. It is very important to organize student conceptual misunderstandings and re-educate students to correct mathematical thinking (Zwart et al., 2017, 2020, 2021, 2022). Conceptual misunderstandings in mathematics by students are in, Arithmetic, Number sense, Exact verse approximate, Fractions, Magnitude for negative numbers, Order of operations, Powers, Square roots – definition, Square roots – with sums, Simplification/factorization of algebraic expressions, Using the definition of the absolute value function, particularly for negative numbers, inequalities, Expansion of algebraic expressions, Exponential – properties, Exponential functions, Logarithm – Properties, Logarithm – solving equations, Functions, Functions – asymptotes, Translational errors. Some of the difficulties students have on simplification of fractions include: Incorrect cancelling of $\frac{ab+c}{b}$ to obtain $a + c$ and mmisunderstanding of "invert and multiply" rule for dividing fractions.

Egodawatte (2011) also found that, most students have difficulty in identifying like terms in algebraic fractions simplification. Example of wrong simplification carried out was when students simplifying $\frac{xa+xb}{x+xd}$ was $\frac{xa+xb}{x+xd} = \frac{a+b}{d}$, Instead of $\frac{xa+xb}{x+xd} = \frac{x(a+b)}{x(1+d)} = \frac{a+b}{1+b}$. According to Egodawatte, the wrong simplification was as a result of conceptual misunderstanding students have on algebraic fractions.

Mangrosi et al (2014) researched students' conception of algebraic properties and their effect on performance in algebra. Their research aimed at investigating the conception of senior high school students of selected public schools in the province of Maguindanao and Lanao del Sur. Their study which used the qualitative-quantitative design on forty-four students as primary participants, one of their findings was that the students showed common patterns of misconceptions in algebraic properties such as misreading and unfamiliarity of terms of the concept in algebra. They concluded that misconception of algebra in like terms identification also contributes to students' poor performance in algebra, hence in mathematics. According to STR (1997), conceptual misunderstandings arise when students are taught scientific information in a way that does not provoke them to confront paradoxes and conflicts resulting from their preconceived notions and non-scientific beliefs. To deal with their confusion, students construct faulty models that usually are so weak that the students themselves are insecure about the concepts. For example, a student might believe that objects float in water because they are lighter than the water.

The West Africa Examination Council (WAEC) chief examiner's report on mathematics performance (2014 and 2015) also shown that students normally make errors which depict some form of conceptual misunderstanding in answering algebraic problems. Examples of such conceptual misunderstanding in simplifying algebraic fraction is: $\left(4\frac{3}{5}-1\frac{5}{6}\right)\div 1\frac{1}{24}\times\left(1\frac{2}{3}+2\frac{1}{2}\right)$.In the simplification, they were not able to convert the mixed fractions into improper fractions and manipulated them correctly.

The observation by the researcher through literature, classroom observation and diagnostic test on errors in algebraic concepts which depict conceptual misunderstanding such as converting mixed fractions to improper fraction ($3\frac{1}{2}=3\times\frac{1}{2}=\frac{3\times 1}{2}=\frac{3}{2}$.), simplification of algebraic expressions and equations was the rational that motivated the researcher to carry out this study.

Methodology

This study used the descriptive survey design to investigate senior high school mathematics teachers, prospective teachers' and students' conceptual misunderstanding in senior high school algebra. Convenience, purposive and simple random sampling were employed as sampling techniques. The sample involves one hundred and ten (110) second-year senior high schools' students, forty (40) SHS mathematics teachers in four selected senior high schools in Kumasi metropolis and Ejisu municipal in Ashanti region of Ghana and sixty (60) final year (level 400) in the Department of Mathematics Education of the University of Education, Winneba (UEW).

The diagnostic algebraic test was the main test instrument for the study. A 2 hours diagnostic algebraic test consisting of 14 items were administered to the one hundred and ten (110) students and the sixty (60)

prospective teachers to answer after which photocopies of the solution scripts were given to the teachers to mark using their prepared marking scheme. The answered test papers from the students and the prospective teachers, and the marking schemes prepared by the teachers were marked and analyzed. The test items and errors found were identified. These errors were scrutinized and analyzed to determine the ones due to conceptual misunderstanding. A sample of five (5) students, three (3) prospective teachers and two (2) teachers were interviewed. The interview was conducted as another means which helped in confirmation of an error as a conceptual misunderstanding or carelessness (mistake/ slip). The findings were presented and discussed.

Findings

The analysis revealed seven categories of conceptual misunderstanding that were prevalent among teachers, prospective teachers and students. These were

1. Recognizing or thinking of like terms as different terms in tasks involving simplification of algebraic fractions

The analysis revealed that 28% of the prospective teachers and 48% of the students depicted this form of conceptual misunderstanding on task 1 and 2. Tasks 1 and 2 of the tests (see Box 1) required the participants to recognize that the terms in the algebraic fractions (terms in both the numerator and denominator) are the same, hence in their simplification they only have to cancel out to obtain the answer 1.

$$(1)\quad \frac{2x}{2x} = \frac{\cancel{2x}}{\cancel{2x}} = 1 \qquad\qquad (2)\quad \frac{2+x}{2+x} = \frac{\cancel{2+x}}{\cancel{2+x}} = 1$$

The participants who demonstrated conceptual misunderstanding in recognizing or thinking of like terms as different terms in tasks involving simplification of algebraic fractions used different methods to do the tasks. Examples of their solutions and reasons to their procedure;

Participants A wrote. $\frac{2x}{2x} = \frac{2x}{2x} \times \frac{2x}{2x}$ and $\frac{2+x}{2+x} = \frac{2+x}{2+x} \times \frac{2+x}{2+x}$

Interviewer: Can you e*xplain your procedure?*

Participants: Yes *"it is an algebraic fraction, and to simplify it, one needs to rationalize the denominator hence the multiplication of both the numerator and the denominator by 2x in 1 and 2+x in 2."*

Participants B wrote $\frac{2x}{2x} = (2x) \div (2x) = (2x)(-2x) = -4x^2$.

Interviewer: Can you e*xplain your solution?*

Participants: *"The division was changed into multiplication, and then simplified. In changing division into multiplication, the denominator becomes negative"*

Participant C wrote $\frac{2+x}{2+x} = (2+x) \div (2+x) = (2+x)(-2-x) = -(x^2+4x+4)$.

Interviewer: Can you e*xplain your solution?*

Participant: *"The division was changed into multiplication, and then binomial expression was expanded"*

2. Seeing or thinking of algebraic variables as symbols which are only assigned to names or labels or constants

75% of Students, 57% of Prospective teachers and 15% of Teachers of the participants demonstrated this form of conceptual misunderstanding.

The solutions from the participants on task 3 is shown below.

Task 3: $Simplify \quad x\left(\frac{a}{b}\right)$

Participant A wrote $x\left(\frac{a}{b}\right) = \frac{ax}{bx} = \frac{a}{b}$.

Interviewer: Can you e*xplain your answer?*

Participants: *"the bracket can be removed by multiplying through by the x outside the bracket and this is done by multiplied both the numerator and the denominator by the x".*

Other solutions on task 3 were;

Participant B wrote $x\left(\frac{a}{b}\right) = \frac{xb+a}{b}$.

Interviewer: Can you e*xplain your answer?*

Response: $x\left(\frac{a}{b}\right)$ *can be simplified by finding the L.C.M for the denominators 1 and b which is b. after finding the LCM you continue to the simplification to obtain* $\left(\frac{xb+a}{b}\right)$

Participant C wrote $x\left(\frac{a}{b}\right)=\frac{a^x}{b^x}$.

Interviewer: Can you *explain your answer?*

Response C: $x\left(\frac{a}{b}\right)$ *is the same as* $\left(\frac{a}{b}\right)^x$ *so removing the bracket will be expanded as* $\frac{a^x}{b^x}$

Participant D and E wrote $x\left(\frac{a}{b}\right)=\frac{ax}{b}$ and then simplified it further as $\frac{ax}{b} \Rightarrow x=\frac{a}{b}$.

Interviewer: Can you *explain your solution?*

Response: I compared left hand side and right-hand side of the equation $x\left(\frac{a}{b}\right)=\left(\frac{ax}{b}\right)$ to obtain $x=\frac{a}{b}$.

3. Seeing or thinking of algebraic variables as symbols which are only assigned to names or labels or constants

77% of Students, 65% of Prospective teachers and 15% of Teachers showed this for of conceptual misunderstanding after the analysis.

Tasks 4, and 5 of the test required the participants to recognize that the variables in the tasks represent a varying quantity or a unit. Some participants had them wrong largely because they think of algebraic variables as symbols which are only assigned to names. Example were;

Task 4: Bernice sells x *oranges*, Priscilla sells three times as many oranges as Bernice. An orange cost $GH₵0.25$. If the oranges are of the same size; *a)* how many variables can be formed from this problem? b) give the name/s of the variable

Participant wrote $Bernice = x \text{ and } Priscilla = y$

Interviewer: Can you *explain your answer?*

Response: "*x should represent the name Bernice while y represents the name Priscilla for names in a question should take variables in other to simplify*"

Task 5: A shirt cost c cedis each and a pair of shoe cost d cedis if Mr. Appiah buys 5 shirts and 4 pairs of shoes. Explain what $5c+4d$ means or represent? And simplify further if possible

Participant wrote $c = shirt,\ d = shoes\ hence,\ 5c+4d = 9cd$

Interviewer: Can you *explain your answer?*

Response: *"9cd means 9 items bought comprising of shirts and shoes"*

Other solutions include;

Participant wrote: $5c + 4d = 9(c + d)$.

Interviewer: Can you *explain how you arrive at this answer?*

Participant: *"$5c + 4d$ can be simplified further by first grouping like terms. So the numbers part were grouped by adding 4 to 5 (4+5) to get 9 and the variables were also grouped by adding c and d to get $c + d$, but $c + d$ cannot be simplified further because they are different variables and it cannot be added".*

Participant wrote: $5c + 4d = 9cd$.

Interviewer: Can you *explain your answer?*

Participant: *I added the numbers 4+5 to get 9 and simplify $c + d$ further to obtained cd because adding c and d will give you cd*

4. Using arithmetic reasoning in mathematizing algebraic expression.

63% of Students, 52% of Prospective teachers and 15% of Teachers see mathematical equation as similar to mathematical expression.

In these tasks (6, 7 and 8) the participants are required to, expand or explain or form a mathematical expression to arrive at correct solution.

Task 6: Explaining what xy means

Participant wrote: $xy \text{ is an equation}$

Interviewer: Can you *explain your answer?*

Response; *"because there are two different variables involve. That is whenever there are two variables irrespective of the sign (+, −, ×, ÷) between them it is an equation".*

Task 7: Expand and simplify $(P - Q)^2$ if possible

Participant wrote $(P - Q)^2 = (P - Q)(P - Q) = 0 \Rightarrow (P - Q) = 0 \therefore P = Q$

Interviewer: Can you e*xplain your answer?*

Participant: *"$(P-Q)^2$ is the same as $(P-Q)^2=0$*

After equating it to zero, I expanded the LHS and continued the simplification"

Other Solutions were:

Task 8: Kofi's age is subtracted from ten and the result is multiplied by two. Write the expression for the statement

Participant wrote $2x-10=x-2 \Rightarrow x=8$

Participant wrote $10-x=2x \Rightarrow x=\frac{10}{3}$

Participant wrote $10-x=0 \Rightarrow x=10$

Interviewer: Can you e*xplain your answer?*

Participants: *To find the age an equation must be formed*

5. Using arithmetic reasoning in interpreting algebraic expression.

90% of the students, 84% of the Prospective Teachers and 20% of the Teachers used arithmetic reasoning in mathematizing algebraic expression.

Tasks 9 and 10 required the participants to recognize the relational relationship and equivalency in variables when mathematizing word problem. Those with wrong solutions largely because they use arithmetic reasoning in mathematizing algebraic expressions. Example of such solutions were;

Task 9: Mr. Asare shared some money to his two sons and a daughter, Ben, John and Mary. Mary received 5 times the amount than Ben, and 4 less than John received. The amount received by Ben and John is GhȻ 22.00. How much did Mr. Asare gives to each child

Participant wrote: *Ben*; *John*; *Mary* in a ratio of $x:5x-4:5x$

Interviewer: Can you e*xplain your answer?*

Participants: *"the cue word "sharing" calls for the use of the concept ratio and proportion and there are three people sharing money, hence the ratios".*

Task 10: Mr. Adu bought 8 books and 12 pens from a shop. A book cost him GhȻ 0.50 more than a pen. If he spent GHȻ 94 altogether, how much did a book and a pen cost

Participant wrote: “More than (>), hence”$8x > 12$

Interviewer: Can you e*xplain your answer?*

Participants: *“The symbol was from, the cue word ‘more than’ because ‘more than’ symbolically represent " >” in mathematics”*

6. Using arithmetic reasoning in interpreting algebraic expression.

85% of the students, 61% of the Prospective Teachers and 7.5% of the Teachers used arithmetic reasoning in interpreting algebraic expression.

Tasks 11 and 12 required the participants to recognize the relational relationship and equivalency in interpreting algebraic expression. Those with wrong solution largely because they use arithmetic reasoning in interpreting algebraic expressions. Examples of their solutions due to this conceptual misunderstanding includes the following;

Task 11: When a number is subtracted from six, the result is two times five less than the number. Find the number

Participants wrote: $let\ the\ number\ be\ x, hence\ x - 6 = 2 \times 5 < x$

Interviewer: Can you e*xplain your answer?*

Participants: *“The word problem was translated base on some cue words (like less than;<) in the statements and according to the order these words appear in the statement”*

Task 12: The letter n represent a natural number given $\frac{1}{n}$ and $\frac{1}{n-1}$ which one is more? Explain your answer

Participants wrote $\frac{1}{n} > \frac{1}{n-1}$; $becuase\ n > n - 1$

Interviewer: Can you e*xplain your answer?*

Participants:.*” is because n being a natural number say 8* $(n = 8)$*, is greater than* $n - 1$ *(8-1=7)”*

7. Seeing or thinking an equal sign as only a step marker to indicate the next step of procedure.

81% of the students, 45% of the Prospective Teacher and 35% of the Teachers sees equal sign as only a step marker to indicate the next step of procedure.

Tasks 13 and 14 required the participants to recognize the equivalence property of the equal sign in simplification. Those with wrong solution largely because they see the equal sign as a step maker to indicate the next step of procedure.

Task 13: $a = \sqrt[3]{\frac{y}{5}} = \left(\sqrt[3]{\frac{y}{5}}\right)^3 = \frac{y}{5}, \ \therefore y = 5a$

Interviewer: Can you e*xplain your answer?*

Participants: *"in solving, the cube root at the RHS was remove by raising it to the power of 3"*.

Task 14: Find the value of *x* $if \sqrt{(x^2+9)} = 5$

Participant wrote $5 = \sqrt{x^2 + 3^2}$

$5 = (\sqrt{(x+3)(x-3)}\ )^2$

5$= (x+3)(x-3) \Longrightarrow x = 3,\ x = 8$

Interviewer: Can you e*xplain your answer?*

Participants: *"solving for the variable, I removed the square root sign at the RHS of the equation by squaring that side and continued with the simplification".*

Discussion

The algebraic tasks 1 and 2 required the participants to recognize that the terms in the algebraic fractions (terms in both the numerator and denominator) are the same, hence in their simplification they only have to cancel out to obtain the answer 1. Most of the students and the prospective teachers in their simplification recognize that the like terms in the algebraic fraction were different and this conceptual misunderstanding led to the use of

different methods which were wrong in the simplification of tasks. Some participants used rationalization of denominator in their simplification whiles others employed domain determination for algebraic fractions and concept of modulo arithmetic as well as other forms of simplifications which were wrong methods on these tasks. The solution process and their reasons given based on the interview revealed their conceptual misunderstanding on these concepts. Example was rewriting of $\frac{2+x}{2+x}$ **as** $\frac{2}{2}+\frac{x}{x}$ was from $\mathbf{3}\frac{2}{5} = \mathbf{3}+\frac{2}{5}$. All these wrong simplification methods used by the participants on the algebraic fraction with like terms was as result of incorrect separation of terms in algebraic expression (Matz, 1980; Allen, 2007; Egodawatte (2011).

Task 3 which required the participants to recognize that the variable *x* is of denominator one, hence in their simplification they only have to multiply the *x* by the numerator *a* to get the correct answer. The wrong method used in their working was due to the conceptual misunderstanding of not recognizing that the denominator of the variable *x* is 1. For correct solution one need to apply the concept of multiplying two fractions where by the numerators and denominators are multiplied separately. In the inquiry to the process in the simplification as $x\left(\frac{a}{b}\right)=\frac{ax}{bx}=\frac{a}{b}$, It was observed that the variable x is taken as a scalar and $\frac{a}{b}$ as a vector, so the concept of scalar multiplication of vector was wrongly applied to this context. This happened because these participants were not able to identify the difference between (a,b) and $\frac{a}{b}$. According to Egodawatte (2011), when algebraic fraction has to be multiplied by a variable, students often use cross multiplication although it is not appropriate.

The simplification of tasks 4 and 5 required the students, prospective teachers and the teachers to recognize that the variable in these tasks represent a varying quantity or a unit but wrong methods were employed due to their conceptual misunderstanding of seeing algebraic variables as symbols which are only assigned to names, or labels or constants. The simplification made on task 5 as $\mathbf{5}c+\mathbf{4}d=\mathbf{9}(c+d)$ **and** $\mathbf{5}c+\mathbf{4}d=\mathbf{9}cd$ by some of the participants reveal that they may have drawn on previous knowledge or other subjects that do not differentiate between joining and adding such as adding oxygen to carbon gives CO_2 in chemistry (Kurian,1990). Also, there is a conceptual misunderstanding that expression $c+d$ or $x+y$ cannot be the final answer hence the need to simplify the expression further. Thus *c+d* as an incomplete answer (Booth,1988). Also, the letters *x* and *y* were assigned to the names Bernice and Priscilla whiles *c* and *d* were also assign to shirt and shoe because a variable can only represent a label or name or thing but not a task.

The wrong solutions to tasks 6, 7 and 8 by the participants was as a result of the conceptual misunderstanding of seeing mathematical expressions as similar to mathematical equations. The solutions "*xy being an equation"* and the simplification $(P-Q)^2=(P-Q)(P-Q)=0 \Rightarrow (P-Q)=0 \therefore P=Q$ shows the difficulty in differentiating between algebraic equation and algebraic expression stated by Allen (2007). The explanation on task 8 that *"the equations were formed because it is the only way to evaluate the variable''* depict their conceptual misunderstanding on equation and expression.

The fifth conceptual misunderstanding identified that 90% of the students, 84% of the prospective teachers and 20% of the teachers used arithmetic reasoning to mathematize algebraic expressions when answering tasks 9 and 10. On this conceptual misunderstanding, most of the reasons by the participants to their solutions were based on some words as cue in the mathematizing algebraic expression and more of these cue words in their mathematizing was incorrect. For example, five times a number is eight more than the number, find the number? Stating $5x > 8$ because of the cue word "more than" is incorrect arithmetic reasoning. The sixth conceptual misunderstanding identified was using arithmetic reasoning in interpreting algebraic expression on tasks 11 and 12. 85% of the students, 61% of the prospective teachers and 10 % of the teachers used arithmetic reasoning in interpreting algebraic expression. Some of the reasons for their answers were as a result of static and syntactic translation (Clement, 1982) and assigning of mathematical symbols to some words. The seventh conceptual misunderstanding identified was seeing or thinking the equal sign as only a step marker to indicate the next step of procedure on tasks 13 and 14. 81% of the students, 45% of the prospective teachers and 35% of the teachers were not able to get correct solution because they did not see the equivalence property of the equal sign, hence in simplifying this task only one side of the equation was considered. This suggests the aspects of arithmetic instruction which is contributing to their difficulties in algebra.

Conclusion

It can be concluded based on the findings from this study that senior high school teachers, prospective teachers and student have some conceptual misunderstanding on senior high school algebra. Six (6) out of the seven (7) conceptual misunderstanding identified among the students and the prospective mathematics teacher were with the in-service teachers. The observed consistency between students', prospective teachers and teachers' conceptual misunderstanding is likely to influence the teaching that many students experience in learning algebra. The results of this study indicated that the prospective teachers and some teachers' conceptual understanding of algebra is not good enough to assist learners to understand algebra. The ability to teach and apply their algebraic concepts clearly without conceptual misunderstanding was limited.

The study consistently indicate that conceptual misunderstanding is deeply-seated and not easily removed. In many instances, learners appear to overcome a conceptual misunderstanding only to have the same conceptual misunderstanding reappear later. This is probably a result of the fact that, when individuals construct learning, they become attached to the notions they have constructed.

Recommendations

One important requirement in eliminating those conceptual misunderstandings is that learners must actively participate in the process of overcoming their conceptual misunderstanding. For the conceptual misunderstanding to be eliminated completely, it is essential that in teaching algebraic concepts, teachers should

be aware of possible conceptual misunderstandings student may have and also provide students with classroom learning environments that help them develop both conceptual and procedural knowledge so that they construct correct conceptions right from the start to the end of the concept.

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